Roll No. .....

(05/25)

15236

M. Sc. (2 Year) EXAMINATION

(For Batch 2021 & Onwards)

(Fourth Semester)

MATHEMATICS

MSc/Maths/4/DSC16

Boundary Value Problems

Time : Three Hours *Maximum Marks :* 70

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (i) The boundary value problem is always converted into.....equation. 2

   (ii) Define *three* types of boundary conditions. 2

(iii) Solve the integral equation :

$$g(s) = f(s) + \int_0^s (s-t)\, g(t)\, dt,$$

using Laplace transform. 2

(iv) Give shifting property of Dirae-Delta functions. 2

(v) Explain basic procedure of perturbation methods for the solution of : 2

$$f(\mathrm{P}) = \int_{\mathrm{S}} \mathrm{K}(\mathrm{P}, \mathrm{Q})\, g(\mathrm{Q})\, ds, \mathrm{P} \in \mathrm{S}$$

Section I

2. (a) Solve the problem : 8

$$\frac{d}{ds}\left(p(s)\frac{dy}{ds}\right) + q(s)y(s) = \mathrm{F}(s)$$

$$y(a) = 0,\ y'(a) = 0.$$

(b) Convert the problem : 7

$$y'' + y = 0,\ y(0) = 0,\ y'(0) = 1$$

into an integral equation.

3. Explain the Modified-Green's function with its properties and use this function to solve the problem : 15

$$-\left(\frac{d^2y}{ds^2}+\lambda y\right)=\mathrm{F}(s)$$

$$y'(0) = y'(e) = 0,\ 0 \leq s \leq l.$$

Section II

4. (a) Define Interior Neumann Problem and find at integral representation formula. 7

(b) Solve the boundary value problem : 8

$$-\nabla^2 u(x)=4\pi\delta(x),\ x \in \mathbf{R},\ u/s=f \text{ using}$$

$$-\nabla^2 \mathrm{G}=\delta(x-\xi),\ \mathrm{G}/s=0$$

5. Find the electorostatic potential of a conducting disk bounded by two parallel planes, $z = b$ and $z = -c$, $b, c > 0$, then boundary value problem is given by :

$$\nabla^2 v\,(\rho, \phi, z)=0$$

in D region.

$$v(v(\delta, \phi, 0) = f^{(n)}(\delta) \cos n\phi, \; 0 \leq \delta \leq a$$

$$v(\delta, \phi, z) = 0 .$$ 15

Section III

6. (a) Using Laplace Transform find the solution of the equation : 8

$$f(s) = \int_0^S k(s^2 - t^2)\, g(t)\, dt, \; s > 0$$

(b) Define the finite Hilbert-transform pair and derive the second form pair of it. 7

7. Define Three-part boundary value problem and find the solution. 15

Section IV

8. (a) Consider the Fredholm integral equation of the first kind :

$$f(\rho) = \int_S K(P, Q)\, g(Q)\, ds, \; P \in S$$

where $K(P, Q) = \frac{e^{ier}}{r}$, ε is perturbation parameter. Find the solution. 8

(b) Solve the boundary value problem : 8

$$\nabla^2 k^2 u_s + k^2 u_s = 0, x \in R_e$$

$$u_s = -u_i \text{ on S.}$$

9. Find the torque experienced by a sphere which is rotating uniformly in oseen flow and is bounded by a pair of parallel walls $z = \pm C$. Also evaluate the velocity field. 15