

20th
EDITION

Ordinary and Partial

Differential Equations

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PART I ELEMENTARY DIFFERENTIAL EQUATIONS

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