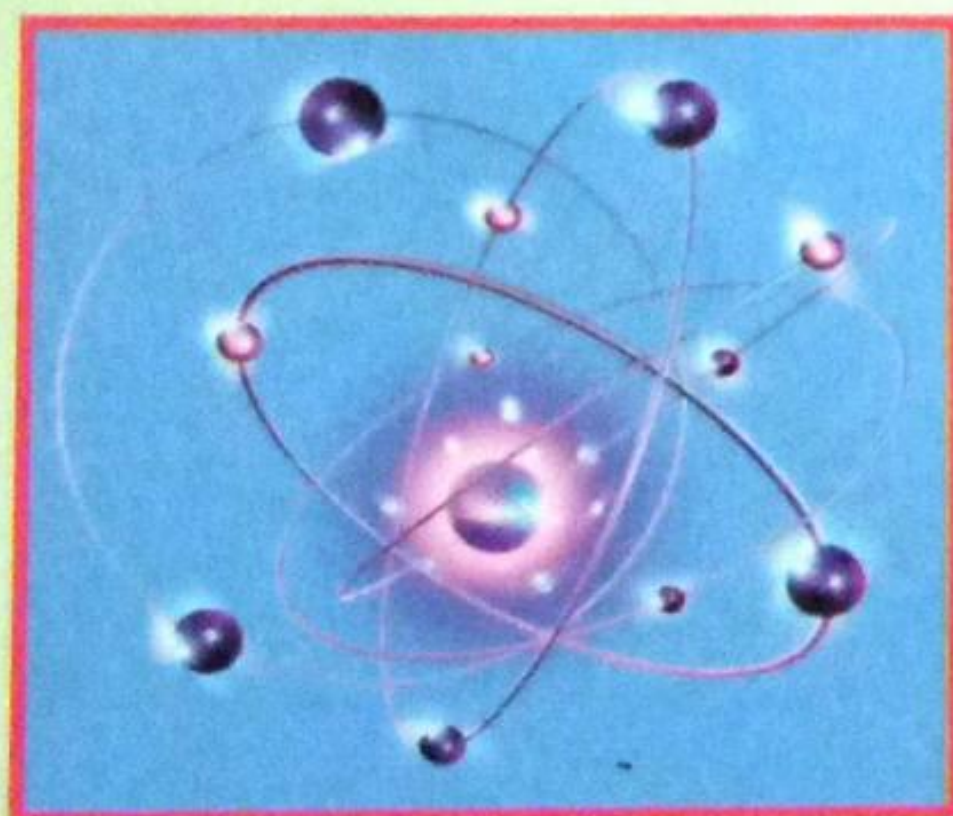


NEW AGE

FOURTH REVISED EDITION

QUANTUM CHEMISTRY



R K Prasad



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