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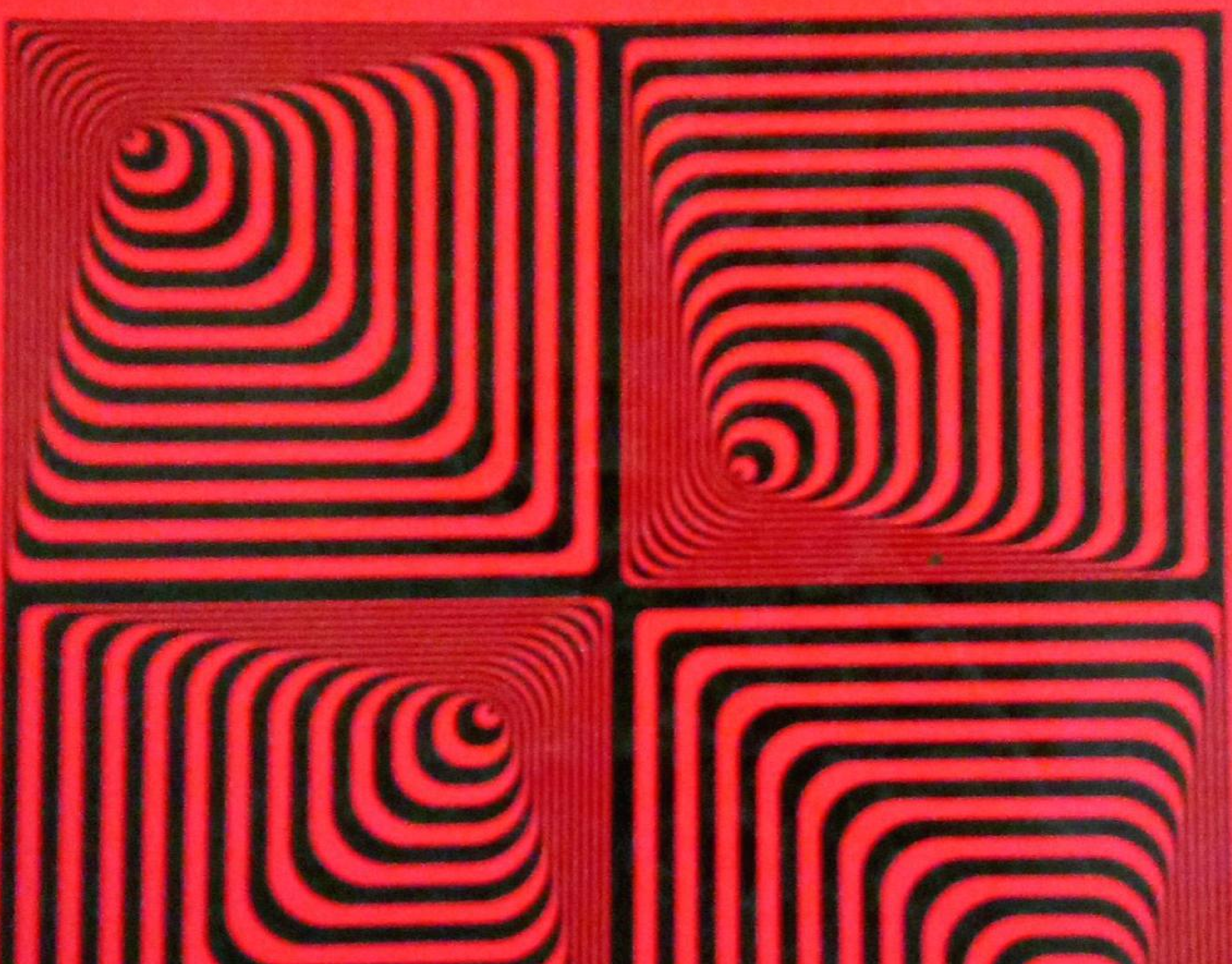
REAL ANALYSIS

FOURTH EDITION



H. L. ROYDEN | P. M. FITZPATRICK

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