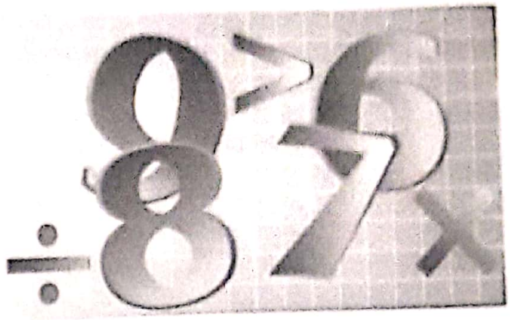
1 Rational Numbers



In our previous class we have studied about natural numbers, whole numbers, integers and fractions. We have also studied about various operations on rational numbers. In this chapter we shall study the properties of these operations on rational numbers.

Rational numbers *The numbers of the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called rational numbers.*

EXAMPLES *Each of the numbers* $\frac{5}{8}, \frac{-3}{14}, \frac{7}{-15}$ *and* $\frac{-6}{-11}$ *is a rational number.*

Positive rationals *A rational number is said to be positive if its numerator and denominator are either both positive or both negative.*

Thus, $\frac{5}{7}$ and $\frac{-2}{-3}$ are both positive rationals.

Negative rationals *A rational number is said to be negative if its numerator and denominator are of opposite signs.*

Thus, $\frac{-4}{9}$ and $\frac{5}{-12}$ are both negative rationals.

Three Properties of Rational Numbers:

Property 1. *If $\frac{a}{b}$ is a rational number and m is a nonzero integer then* $\frac{a}{b} = \frac{a \times m}{b \times m}$.

EXAMPLE $\frac{-3}{4} = \frac{(-3) \times 2}{4 \times 2} = \frac{(-3) \times 3}{4 \times 3} = \frac{(-3) \times 4}{4 \times 4} = \ldots$

$\Rightarrow \frac{-3}{4} = \frac{-6}{8} = \frac{-9}{12} = \frac{-12}{16} = \ldots$

Such rational numbers are called ***equivalent rational numbers.***

Property 2. *If $\frac{a}{b}$ is a rational number and m is a common divisor of a and b, then* $\frac{a}{b} = \frac{a \div m}{b \div m}$.

Thus, we can write, $\frac{-32}{40} = \frac{-32 \div 8}{40 \div 8} = \frac{-4}{5}$.

Standard form of a rational number

A rational number $\frac{a}{b}$ is said to be in standard form if a and b are integers having no common divisor other than 1 and b is positive.

EXAMPLE 1. *Express $\frac{33}{-44}$ in standard form.*

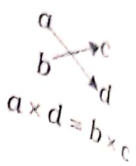
Solution $\frac{33}{-44} = \frac{33 \times (-1)}{(-44) \times (-1)} = \frac{-33}{44}$

The greatest common divisor of 33 and 44 is 11.

$\therefore \frac{-33}{44} = \frac{(-33) \div 11}{44 \div 11} = \frac{-3}{4}$.

Hence, $\frac{-33}{44} = \frac{-3}{4}$ (in standard form).

Property 3. *Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers. Then,* $\frac{a}{b} = \frac{c}{d} \Leftrightarrow (a \times d) = (b \times c)$.



COMPARISON OF RATIONAL NUMBERS

It is clear that:

(i) every positive rational number is greater than 0.
(ii) every negative rational number is less than 0.

GENERAL METHOD OF COMPARING RATIONAL NUMBERS

Step 1: Express each of the two given rational numbers with positive denominator.

Step 2: Take the LCM of these positive denominators.

Step 3: Express each rational number (obtained in Step 1) with this LCM as the common denominator.

Step 4: The number having the greater numerator is greater.

EXAMPLE 2. *Which of the numbers $\frac{3}{-4}$ or $\frac{-5}{6}$ is greater?*

Solution First we write each of the given numbers with positive denominator.

One number $= \frac{3}{-4} = \frac{3 \times (-1)}{(-4) \times (-1)} = \frac{-3}{4}$.

The other number $= \frac{-5}{6}$.

LCM of 4 and 6 = 12.

$\therefore \frac{-3}{4} = \frac{(-3) \times 3}{4 \times 3} = \frac{-9}{12}$ and $\frac{-5}{6} = \frac{(-5) \times 2}{6 \times 2} = \frac{-10}{12}$.

Clearly, $-9 > -10$. $\therefore \frac{-9}{12} > \frac{-10}{12}$.

Hence, $\frac{-3}{4} > \frac{-5}{6}$, i.e., $\frac{3}{-4} > \frac{-5}{6}$.

EXAMPLE 3. *Arrange the numbers $\frac{-3}{5}, \frac{7}{-10}$ and $\frac{-5}{8}$ in ascending order.*

Solution First we write each of the given numbers with positive denominator. We have:

$\frac{7}{-10} = \frac{7 \times (-1)}{(-10) \times (-1)} = \frac{-7}{10}$.

Thus, the given numbers are $\frac{-3}{5}, \frac{-7}{10}$ and $\frac{-5}{8}$.

LCM of 5, 10 and 8 is 40.

Now, $\frac{-3}{5} = \frac{(-3)\times 8}{5\times 8} = \frac{-24}{40}$; $\frac{-7}{10} = \frac{(-7)\times 4}{10\times 4} = \frac{-28}{40}$ and $\frac{-5}{8} = \frac{(-5)\times 5}{8\times 5} = \frac{-25}{40}$

Clearly, $\frac{-28}{40} < \frac{-25}{40} < \frac{-24}{40}$.

Hence, $\frac{-7}{10} < \frac{-5}{8} < \frac{-3}{5}$, i.e., $\frac{7}{-10} < \frac{-5}{8} < \frac{-3}{5}$.

EXERCISE 1A

1. Express $\frac{-3}{5}$ as a rational number with denominator

   (i) 20 (ii) −30 (iii) 35 (iv) −40

2. Express $\frac{-42}{98}$ as a rational number with denominator 7.

3. Express $\frac{-48}{60}$ as a rational number with denominator 5.

4. Express each of the following rational numbers in standard form:

   (i) $\frac{-12}{30}$ (ii) $\frac{-14}{49}$ (iii) $\frac{24}{-64}$ (iv) $\frac{-36}{-63}$

5. Which of the two rational numbers is greater in the given pair?

   (i) $\frac{3}{8}$ or 0 (ii) $\frac{-2}{9}$ or 0 (iii) $\frac{-3}{4}$ or $\frac{1}{4}$

   (iv) $\frac{-5}{7}$ or $\frac{-4}{7}$ (v) $\frac{2}{3}$ or $\frac{3}{4}$ (vi) $\frac{-1}{2}$ or −1

6. Which of the two rational numbers is greater in the given pair?

   (i) $\frac{-4}{3}$ or $\frac{-8}{7}$ (ii) $\frac{7}{-9}$ or $\frac{-5}{8}$ (iii) $\frac{-1}{3}$ or $\frac{4}{-5}$

   (iv) $\frac{9}{-13}$ or $\frac{7}{-12}$ (v) $\frac{4}{-5}$ or $\frac{-7}{10}$ (vi) $\frac{-12}{5}$ or −3

7. Fill in the blanks with the correct symbol out of >, = and <:

   (i) $\frac{-3}{7} \ldots\ldots \frac{6}{-13}$ (ii) $\frac{5}{-13} \ldots\ldots \frac{-35}{91}$ (iii) $-2 \ldots\ldots \frac{-13}{5}$

   (iv) $\frac{-2}{3} \ldots\ldots \frac{5}{-8}$ (v) $0 \ldots\ldots \frac{-3}{-5}$ (vi) $\frac{-8}{9} \ldots\ldots \frac{-9}{10}$

8. Arrange the following rational numbers in ascending order:

   (i) $\frac{4}{-9}, \frac{-5}{12}, \frac{7}{-18}, \frac{-2}{3}$ (ii) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$

   (iii) $\frac{3}{-5}, \frac{-7}{10}, \frac{-11}{15}, \frac{-13}{20}$ (iv) $\frac{-4}{7}, \frac{-9}{14}, \frac{13}{-28}, \frac{-23}{42}$

9. Arrange the following rational numbers in descending order:

   (i) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$ (ii) $\frac{-3}{10}, \frac{7}{-15}, \frac{-11}{20}, \frac{17}{-30}$

   (iii) $\frac{-5}{6}, \frac{-7}{12}, \frac{-13}{18}, \frac{23}{-24}$ (iv) $\frac{-10}{11}, \frac{-19}{22}, \frac{-23}{33}, \frac{-39}{44}$

10. Which of the following statements are true and which are false?

    (i) Every whole number is a rational number.

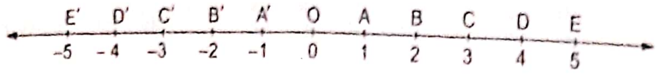
    (ii) Every integer is a rational number.

(iii) 0 is a whole number but it is not a rational number.

REPRESENTATION OF RATIONAL NUMBERS ON THE REAL LINE

In the previous class we have learnt how to represent integers on the number line. Let us review it.

Draw any line. Take a point O on it. Call it 0 (zero). Set off equal distances on the right as well as on the left of O. Such a distance is known as a unit length. Clearly, the points A, B, C, D and E represent the integers 1, 2, 3, 4 and 5 respectively and the points A', B', C', D' and E' represent the integers −1, −2, −3, −4 and −5 respectively.



Thus, we may represent any integer by a point on the number line. Clearly, every positive integer lies to the right of O and every negative integer lies to the left of O.

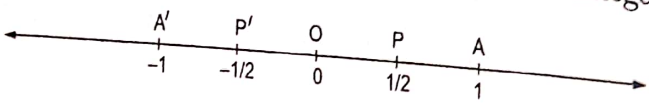
Similarly we can represent rational numbers.

Consider the following examples.

EXAMPLE 1. *Represent $\frac{1}{2}$ and $-\frac{1}{2}$ on the number line.*

Solution Draw a line. Take a point O on it . Let it represent 0. Set off unit lengths OA and OA' to the right and to the left of O respectively.

Then, A represents the integer 1 and A' represents the integer −1.



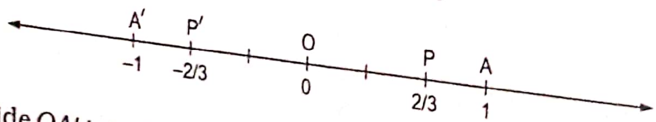
Now, divide OA into two equal parts. Let OP be the first part out of these two parts. Then, the point P represents the rational number $\frac{1}{2}$.

Again, divide OA' into two equal parts. Let OP' be the first part out of these 2 parts. Then, the point P' represents the rational number $-\frac{1}{2}$.

EXAMPLE 2. *Represent $\frac{2}{3}$ and $-\frac{2}{3}$ on the number line.*

Solution Draw a line. Take a point O on it. Let it represent 0. From O set off unit distances OA and OA' to the right and left of O respectively.

Divide OA into 3 equal parts. Let OP be the segment showing 2 parts out of 3. Then, the point P represents the rational number $\frac{2}{3}$.



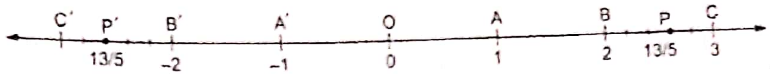
Again divide OA' into 3 equal parts. Let OP' be the segment consisting of 2 parts out of these 3 parts. Then, the point P' represents the rational number $-\frac{2}{3}$.

EXAMPLE 3. *Represent $\frac{13}{5}$ and $\frac{-13}{5}$ on the number line.*

Solution Draw a line. Take a point O on it. Let it represent 0.

Now, $\frac{13}{5} = 2\frac{3}{5} = 2 + \frac{3}{5}$.

From O, set off unit distances OA, AB and BC to the right of O. Clearly, the points A, B and C represent the integers 1, 2 and 3 respectively. Now, take 2 units OA and AB, and divide the third unit BC into 5 equal parts. Take 3 parts out of these 5 parts to reach at a point P. Then, the point P represents the rational number $\frac{13}{5}$.



Again, from O, set off unit distances to the left. Let these segments be OA', $A'B'$, $B'C'$, etc. Then, clearly the points A', B' and C' represent the integers -1, -2, -3 respectively.

Now, $\frac{-13}{5} = -\left(2 + \frac{3}{5}\right)$.

Take 2 full unit lengths to the left of O. Divide the third unit $B'C'$ into 5 equal parts. Take 3 parts out of these 5 parts to reach a point P'.

Then, the point P' represents the rational number $\frac{-13}{5}$.

Thus, we can represent every rational number by a point on the number line.

EXERCISE 1B

1. Represent each of the following numbers on the number line:

   (i) $\frac{1}{3}$ (ii) $\frac{2}{7}$ (iii) $1\frac{3}{4}$ (iv) $2\frac{2}{5}$

   (v) $3\frac{1}{2}$ (vi) $5\frac{5}{7}$ (vii) $4\frac{2}{3}$ (viii) 8

2. Represent each of the following numbers on the number line:

   (i) $\frac{-1}{3}$ (ii) $\frac{-3}{4}$ (iii) $-1\frac{2}{3}$ (iv) $-3\frac{1}{7}$

   (v) $-4\frac{3}{5}$ (vi) $-2\frac{5}{6}$ (vii) -3 (viii) $-2\frac{7}{8}$

3. Which of the following statements are true and which are false?

   (i) $\frac{-3}{5}$ lies to the left of 0 on the number line.

   (ii) $\frac{-12}{7}$ lies to the right of 0 on the number line.

   (iii) The rational numbers $\frac{1}{3}$ and $\frac{-5}{2}$ are on opposite sides of 0 on the number line.

   (iv) The rational number $\frac{-18}{-13}$ lies to the left of 0 on the number line.

ADDITION OF RATIONAL NUMBERS

If two rational numbers are to be added, we should convert each of them into a rational number with positive denominator.

CASE 1 **When Given Numbers have Same Denominator:**

In this case, we define $\left(\frac{a}{b}+\frac{c}{b}\right)=\frac{(a+c)}{b}$

EXAMPLE 1 *Find the sum:*

(i) $\frac{7}{9}+\frac{-11}{9}$ (ii) $\frac{8}{-11}+\frac{3}{11}$

Solution We have:

(i) $\frac{7}{9}+\frac{-11}{9}=\frac{7+(-11)}{9}=\frac{-4}{9}$

(ii) $\frac{8}{-11}=\frac{8\times(-1)}{(-11)\times(-1)}=\frac{-8}{11}$

$\therefore \left(\frac{8}{-11}+\frac{3}{11}\right)=\left(\frac{-8}{11}+\frac{3}{11}\right)=\frac{(-8)+3}{11}=\frac{-5}{11}$

CASE 2 **When Denominators of Given Numbers are Unequal:**

Method *In this case we take the LCM of their denominators and express each of the given numbers with this LCM as the common denominator. Now, we add these numbers as shown above.*

EXAMPLE 2 *Find the sum:* $\frac{-5}{6}+\frac{4}{9}$.

Solution The denominators of the given rational numbers are 6 and 9 respectively.

LCM of 6 and 9 = $(3\times2\times3)=18$.

$$\begin{array}{r|l} 3 & 6,\ 9 \\ \hline & 2,\ 3 \end{array}$$

Now, $\frac{-5}{6}=\frac{(-5)\times3}{6\times3}=\frac{-15}{18}$ and $\frac{4}{9}=\frac{4\times2}{9\times2}=\frac{8}{18}$.

$\therefore \left(\frac{-5}{6}+\frac{4}{9}\right)=\left(\frac{-15}{18}+\frac{8}{18}\right)=\frac{(-15)+8}{18}=\frac{-7}{18}$.

Short-Cut Method

EXAMPLE 3. *Find the sum:* $\frac{-9}{16}+\frac{5}{12}$.

Solution LCM of 16 and 12 = $(4\times4\times3)=48$.

$$\begin{array}{r|l} 4 & 16,\ 12 \\ \hline & 4,\ 3 \end{array}$$

$\therefore \frac{-9}{16}+\frac{5}{12}=\frac{3\times(-9)+4\times5}{48}=\frac{(-27)+20}{48}=\frac{-7}{48}$.

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

Property 1 (Closure Property): *The sum of two rational numbers is always a rational number.*

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

EXAMPLES (i) Consider the rational numbers $\frac{1}{3}$ and $\frac{3}{4}$. Then,

$\left(\frac{1}{3}+\frac{3}{4}\right)=\frac{(4+9)}{12}=\frac{13}{12}$, which is a rational number.

(ii) Consider the rational numbers $\frac{-2}{3}$ and $\frac{4}{5}$. Then,

$$\left(\frac{-2}{3}+\frac{4}{5}\right)=\frac{(-10+12)}{15}=\frac{2}{15},$$ which is a rational number.

(iii) Consider the rational numbers $\frac{-5}{12}$ and $\frac{-1}{4}$. Then,

$$\left(\frac{-5}{12}+\frac{-1}{4}\right)=\frac{\{-5+(-3)\}}{12}=\frac{-8}{12}=\frac{-2}{3},$$ which is a rational number.

Property 2 (Commutative Law): *Two rational numbers can be added in any order.*

Thus for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$$\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right).$$

EXAMPLES (i) $\left(\frac{1}{2}+\frac{3}{4}\right)=\frac{(2+3)}{4}=\frac{5}{4}$ and $\left(\frac{3}{4}+\frac{1}{2}\right)=\frac{(3+2)}{4}=\frac{5}{4}$.

$\therefore \left(\frac{1}{2}+\frac{3}{4}\right)=\left(\frac{3}{4}+\frac{1}{2}\right)$.

(ii) $\left\{\frac{3}{8}+\frac{-5}{6}\right\}=\frac{\{9+(-20)\}}{24}=\frac{-11}{24}$ and $\left\{\frac{-5}{6}+\frac{3}{8}\right\}=\frac{\{-20+9\}}{24}=\frac{-11}{24}$.

$\therefore \left(\frac{3}{8}+\frac{-5}{6}\right)=\left(\frac{-5}{6}+\frac{3}{8}\right)$.

(iii) $\left(\frac{-1}{2}+\frac{-2}{3}\right)=\frac{\{(-3)+(-4)\}}{6}=\frac{-7}{6}$ and $\left(\frac{-2}{3}+\frac{-1}{2}\right)=\frac{\{(-4)+(-3)\}}{6}=\frac{-7}{6}$.

$\therefore \left(\frac{-1}{2}+\frac{-2}{3}\right)=\left(\frac{-2}{3}+\frac{-1}{2}\right)$.

Property 3 (Associative Law): *While adding three rational numbers, they can be grouped in any order.*

Thus, for any three rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$, we have

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right).$$

EXAMPLE Consider three rationals $\frac{-2}{3}, \frac{5}{7}$ and $\frac{1}{6}$. Then,

$$\left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{(-14+15)}{21}+\frac{1}{6}\right\}=\left(\frac{1}{21}+\frac{1}{6}\right)=\frac{(2+7)}{42}=\frac{9}{42}=\frac{3}{14}$$

and $$\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}=\left\{\frac{-2}{3}+\frac{(30+7)}{42}\right\}=\left(\frac{-2}{3}+\frac{37}{42}\right)=\frac{(-28+37)}{42}=\frac{9}{42}=\frac{3}{14}.$$

$$\therefore \left\{\left(\frac{-2}{3}+\frac{5}{7}\right)+\frac{1}{6}\right\}=\left\{\frac{-2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)\right\}.$$

Property 4 (Existence of Additive Identity): *0 is a rational number such that the sum of any rational number and 0 is the rational number itself.*

Thus, $\left(\frac{a}{b}+0\right)=\left(0+\frac{a}{b}\right)=\frac{a}{b}$, for every rational number $\frac{a}{b}$.

0 is called the ***additive identity*** for rationals.

EXAMPLES (i) $\left(\frac{3}{5}+0\right)=\left(\frac{3}{5}+\frac{0}{5}\right)=\frac{(3+0)}{5}=\frac{3}{5}$ and similarly, $\left(0+\frac{3}{5}\right)=\frac{3}{5}$.

$\therefore \left(\frac{3}{5}+0\right)=\left(0+\frac{3}{5}\right)=\frac{3}{5}$.

(ii) $\left(\frac{-2}{3}+0\right)=\left(\frac{-2}{3}+\frac{0}{3}\right)=\frac{(-2+0)}{3}=\frac{-2}{3}$ and similarly, $\left(0+\frac{-2}{3}\right)=\frac{-2}{3}$.

$\therefore \left(\frac{-2}{3}+0\right)=\left(0+\frac{-2}{3}\right)=\frac{-2}{3}$.

Property 5 (Existence of Additive Inverse): *For every rational number $\frac{a}{b}$, there exists a rational number $\frac{-a}{b}$ such that $\left(\frac{a}{b}+\frac{-a}{b}\right)=\frac{\{a+(-a)\}}{b}=\frac{0}{b}=0$ and similarly, $\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.*

Thus, $\left(\frac{a}{b}+\frac{-a}{b}\right)=\left(\frac{-a}{b}+\frac{a}{b}\right)=0$.

$\frac{-a}{b}$ is called the ***additive inverse*** of $\frac{a}{b}$.

EXAMPLE $\left(\frac{4}{7}+\frac{-4}{7}\right)=\frac{\{4+(-4)\}}{7}=\frac{0}{7}=0$ and similarly, $\left(\frac{-4}{7}+\frac{4}{7}\right)=0$.

$\therefore \left(\frac{4}{7}+\frac{-4}{7}\right)=\left(\frac{-4}{7}+\frac{4}{7}\right)=0$.

Thus, $\frac{4}{7}$ and $\frac{-4}{7}$ are additive inverses of each other.

SUBTRACTION OF RATIONAL NUMBERS

For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we define:

$$\left(\frac{a}{b}-\frac{c}{d}\right)=\frac{a}{b}+\left(\frac{-c}{d}\right)=\frac{a}{b}+\left(\textit{additive inverse of } \frac{c}{d}\right).$$

SOLVED EXAMPLES

EXAMPLE 1. *Find the additive inverse of:*

(i) $\frac{5}{9}$ (ii) $\frac{-15}{8}$ (iii) $\frac{9}{-11}$ (iv) $\frac{-6}{-7}$

Solution (i) Additive inverse of $\frac{5}{9}$ is $\frac{-5}{9}$.

(ii) Additive inverse of $\frac{-15}{8}$ is $\frac{15}{8}$.

(iii) In standard form, we write $\frac{9}{-11}$ as $\frac{-9}{11}$.

Hence, its additive inverse is $\frac{9}{11}$.

(iv) We may write, $\frac{-6}{-7}=\frac{(-6)\times(-1)}{(-7)\times(-1)}=\frac{6}{7}$.

Hence, its additive inverse is $\frac{-6}{7}$.

EXAMPLE 2. *(i) Subtract* $\frac{3}{4}$ *from* $\frac{2}{3}$ *(ii) Subtract* $\frac{-5}{7}$ *from* $\frac{-2}{5}$

Solution (i) $\left(\frac{2}{3}-\frac{3}{4}\right)=\frac{2}{3}+\left(\text{additive inverse of }\frac{3}{4}\right)$

$$=\left(\frac{2}{3}+\frac{-3}{4}\right)=\frac{\{8+(-9)\}}{12}=\frac{-1}{12}.$$

(ii) $\left\{\frac{-2}{5}-\left(\frac{-5}{7}\right)\right\}=\frac{-2}{5}+\left(\text{additive inverse of }\frac{-5}{7}\right)$

$$=\left(\frac{-2}{5}+\frac{5}{7}\right)\quad\left[\because \text{ additive inverse of } \frac{-5}{7} \text{ is } \frac{5}{7}\right]$$

$$=\frac{(-14+25)}{35}=\frac{11}{35}.$$

EXAMPLE 3. *The sum of two rational numbers is* -5. *If one of them is* $\frac{-13}{6}$, *find the other.*

Solution Let the other number be x. Then,

$$x+\left(\frac{-13}{6}\right)=-5 \Rightarrow x=-5+\left(\text{additive inverse of }\frac{-13}{6}\right)$$

$$\Rightarrow x=\left(-5+\frac{13}{6}\right)=\left(\frac{-5}{1}+\frac{13}{6}\right)=\frac{(-30+13)}{6}$$

$$\Rightarrow x=\frac{-17}{6}.$$

Hence, the required number is $\frac{-17}{6}$.

EXAMPLE 4. *What number should be added to* $\frac{-7}{8}$ *to get* $\frac{4}{9}$?

Solution Let the required number to be added be x. Then,

$$\frac{-7}{8}+x=\frac{4}{9} \Rightarrow x=\frac{4}{9}+\left(\text{additive inverse of }\frac{-7}{8}\right)$$

$$\Rightarrow x=\left(\frac{4}{9}+\frac{7}{8}\right)=\frac{(32+63)}{72}=\frac{95}{72}.$$

Hence, the required number is $\frac{95}{72}$.

EXAMPLE 5. *Evaluate* $\frac{3}{5}+\frac{7}{3}+\frac{-11}{5}+\frac{-2}{3}$.

Solution Using the commutative and associative laws, it follows that we may arrange the terms in any manner suitably. Using this rearrangement property, we have:

$$\frac{3}{5}+\frac{7}{3}+\frac{-11}{5}+\frac{-2}{3}=\left(\frac{3}{5}+\frac{-11}{5}\right)+\left(\frac{7}{3}+\frac{-2}{3}\right)$$

$$=\frac{\{3+(-11)\}}{5}+\frac{\{7+(-2)\}}{3}=\frac{-8}{5}+\frac{5}{3}$$

$$=\frac{(-24+25)}{15}=\frac{1}{15}.$$

EXAMPLE 6. *Simplify:* $\left(\frac{4}{7}+\frac{-8}{9}+\frac{-5}{21}+\frac{1}{3}\right)$.

Solution Using the rearrangement property, we have:

$$\frac{4}{7}+\frac{-8}{9}+\frac{-5}{21}+\frac{1}{3}=\left(\frac{4}{7}+\frac{-5}{21}\right)+\left(\frac{-8}{9}+\frac{1}{3}\right)$$

$$=\frac{(12+(-5))}{21}+\frac{(-8+3)}{9}$$

$$=\left(\frac{7}{21}+\frac{-5}{9}\right)=\frac{(21+(-35))}{63}=\frac{-14}{63}=\frac{-2}{9}.$$

EXAMPLE 7 *What should be subtracted from $\frac{-5}{7}$ to get -1?*

Solution Let the required number be x. Then,

$$\frac{-5}{7}-x=-1 \Rightarrow \frac{-5}{7}=x-1$$

$$\Rightarrow x=\left(\frac{-5}{7}+1\right)=\frac{(-5+7)}{7}=\frac{2}{7}.$$

Hence, the required number is $\frac{2}{7}$.

EXERCISE 1C

1. Add the following rational numbers:

(i) $\frac{-2}{5}$ and $\frac{4}{5}$ (ii) $\frac{-6}{11}$ and $\frac{-4}{11}$ (iii) $\frac{-11}{8}$ and $\frac{5}{8}$

(iv) $\frac{-7}{3}$ and $\frac{1}{3}$ (v) $\frac{5}{6}$ and $\frac{-1}{6}$ (vi) $\frac{-17}{15}$ and $\frac{-1}{15}$

2. Add the following rational numbers:

(i) $\frac{3}{4}$ and $\frac{-3}{5}$ (ii) $\frac{5}{8}$ and $\frac{-7}{12}$ (iii) $\frac{-8}{9}$ and $\frac{11}{6}$

(iv) $\frac{-5}{16}$ and $\frac{7}{24}$ (v) $\frac{7}{-18}$ and $\frac{8}{27}$ (vi) $\frac{1}{-12}$ and $\frac{2}{-15}$

(vii) -1 and $\frac{3}{4}$ (viii) 2 and $\frac{-5}{4}$ (ix) 0 and $\frac{-2}{5}$

3. Verify the following:

(i) $\frac{-12}{5}+\frac{2}{7}=\frac{2}{7}+\frac{-12}{5}$ (ii) $\frac{-5}{8}+\frac{-9}{13}=\frac{-9}{13}+\frac{-5}{8}$

(iii) $3+\frac{-7}{12}=\frac{-7}{12}+3$ (iv) $\frac{2}{-7}+\frac{12}{-35}=\frac{12}{-35}+\frac{2}{-7}$

4. Verify the following:

(i) $\left(\frac{3}{4}+\frac{-2}{5}\right)+\frac{-7}{10}=\frac{3}{4}+\left(\frac{-2}{5}+\frac{-7}{10}\right)$ (ii) $\left(\frac{-7}{11}+\frac{2}{-5}\right)+\frac{-13}{22}=\frac{-7}{11}+\left(\frac{2}{-5}+\frac{-13}{22}\right)$

(iii) $-1+\left(\frac{-2}{3}+\frac{-3}{4}\right)=\left(-1+\frac{-2}{3}\right)+\frac{-3}{4}$

5. Fill in the blanks.

(i) $\left(\frac{-3}{17}\right)+\left(\frac{-12}{5}\right)=\left(\frac{-12}{5}\right)+(\ldots\ldots)$ (ii) $-9+\frac{-21}{8}=(\ldots\ldots)+(-9)$

(iii) $\left(\frac{-8}{13}+\frac{3}{7}\right)+\left(\frac{-13}{4}\right)=(\ldots)+\left[\frac{3}{7}+\left(\frac{-13}{4}\right)\right]$

(iv) $-12+\left(\frac{7}{12}+\frac{-9}{11}\right)=\left(-12+\frac{7}{12}\right)+(\ldots)$

(v) $\frac{19}{-5}+\left(\frac{-3}{11}+\frac{-7}{8}\right)=\left\{\frac{19}{-5}+(\ldots)\right\}+\frac{-7}{8}$

(vi) $\frac{-16}{7}+\ldots=\ldots+\frac{-16}{7}=\frac{-16}{7}$

6. Find the additive inverse of each of the following:

(i) $\frac{1}{3}$ (ii) $\frac{23}{9}$ (iii) -18 (iv) $\frac{-17}{8}$ (v) $\frac{15}{-4}$

(vi) $\frac{-16}{-5}$ (vii) $\frac{-3}{11}$ (viii) 0 (ix) $\frac{19}{-6}$ (x) $\frac{-8}{-7}$

7. Subtract:

(i) $\frac{3}{4}$ from $\frac{1}{3}$ (ii) $\frac{-5}{6}$ from $\frac{1}{3}$ (iii) $\frac{-8}{9}$ from $\frac{-3}{5}$ (iv) $\frac{-9}{7}$ from -1

(v) $\frac{-18}{11}$ from 1 (vi) $\frac{-13}{9}$ from 0 (vii) $\frac{-32}{13}$ from $\frac{-6}{5}$ (viii) -7 from $\frac{-4}{7}$

8. Using the rearrangement property find the sum:

(i) $\frac{4}{3}+\frac{3}{5}+\frac{-2}{3}+\frac{-11}{5}$ (ii) $\frac{-8}{3}+\frac{-1}{4}+\frac{-11}{6}+\frac{3}{8}$

(iii) $\frac{-13}{20}+\frac{11}{14}+\frac{-5}{7}+\frac{7}{10}$ (iv) $\frac{-6}{7}+\frac{-5}{6}+\frac{-4}{9}+\frac{-15}{7}$

9. The sum of two rational numbers is -2. If one of the numbers is $\frac{-14}{5}$, find the other.

10. The sum of two rational numbers is $\frac{-1}{2}$. If one of the numbers is $\frac{5}{6}$, find the other.

11. What number should be added to $\frac{-5}{8}$ so as to get $\frac{-3}{2}$?

12. What number should be added to -1 so as to get $\frac{5}{7}$?

13. What number should be subtracted from $\frac{-2}{3}$ to get $\frac{-1}{6}$?

14. (i) Which rational number is its own additive inverse?
(ii) Is the difference of two rational numbers a rational number?
(iii) Is addition commutative on rational numbers?
(iv) Is addition associative on rational numbers?
(v) Is subtraction commutative on rational numbers?
(vi) Is subtraction associative on rational numbers?
(vii) What is the negative of a negative rational number?

MULTIPLICATION OF RATIONAL NUMBERS

For any two rationals $\frac{a}{b}$ and $\frac{c}{d}$, we define:

$$\left(\frac{a}{b}\times\frac{c}{d}\right)=\frac{(a\times c)}{(b\times d)}$$

SOLVED EXAMPLES

EXAMPLE 1 *Find each of the following products:*

(i) $\frac{2}{3}\times\frac{-5}{7}$ (ii) $\frac{-7}{8}\times\frac{3}{5}$ (iii) $\frac{-15}{4}\times\frac{-3}{8}$

Solution We have:

(i) $\frac{2}{3}\times\frac{-5}{7}=\frac{2\times(-5)}{3\times7}=\frac{-10}{21}$.

(ii) $\frac{-7}{8}\times\frac{3}{5}=\frac{(-7)\times3}{8\times5}=\frac{-21}{40}$.

(iii) $\frac{-15}{4}\times\frac{-3}{8}=\frac{(-15)\times(-3)}{4\times8}=\frac{45}{32}$.

EXAMPLE 2 *Find each of the following products:*

(i) $\frac{-3}{7}\times\frac{14}{5}$ (ii) $\frac{13}{6}\times\frac{-18}{91}$ (iii) $\frac{-11}{9}\times\frac{-51}{44}$

Solution We have:

(i) $\frac{-3}{7}\times\frac{14}{5}=\frac{(-3)\times\cancel{14}^{2}}{{}_{1}\cancel{7}\times5}=\frac{-6}{5}$.

(ii) $\frac{13}{6}\times\frac{-18}{91}=\frac{13\times(-18)}{6\times91}=\frac{-(\cancel{13}^{1}\times\cancel{18}^{3})}{(\cancel{6}_{1}\times\cancel{91}_{7})}=\frac{-3}{7}$.

(iii) $\frac{-11}{9}\times\frac{-51}{44}=\frac{(-11)\times(-51)}{9\times44}=\frac{\cancel{11}^{1}\times\cancel{51}^{17}}{\cancel{9}_{3}\times\cancel{44}_{4}}=\frac{17}{12}$.

PROPERTIES OF MULTIPLICATION OF RATIONAL NUMBERS

Property 1 (Closure Property): *The product of two rational numbers is always a rational number.*

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

EXAMPLES (i) Consider the rational numbers $\frac{1}{2}$ and $\frac{5}{7}$. Then,

$\left(\frac{1}{2}\times\frac{5}{7}\right)=\frac{(1\times5)}{(2\times7)}=\frac{5}{14}$, which is a rational number.

(ii) Consider the rational numbers $\frac{-3}{7}$ and $\frac{5}{14}$. Then,

$\left(\frac{-3}{7}\times\frac{5}{14}\right)=\frac{(-3)\times5}{7\times14}=\frac{-15}{98}$, which is a rational number.

(iii) Consider the rational numbers $\frac{-4}{5}$ and $\frac{-7}{3}$. Then,

$\left(\frac{-4}{5}\times\frac{-7}{3}\right)=\frac{(-4)\times(-7)}{5\times3}=\frac{28}{15}$, which is a rational number.

Property 2 (Commutative Law): *Two rational numbers can be multiplied in any order.*

Thus, for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$$

EXAMPLES (i) Let us consider the rational numbers $\frac{3}{4}$ and $\frac{5}{7}$. Then,

$$\left(\frac{3}{4}\times\frac{5}{7}\right)=\frac{(3\times5)}{(4\times7)}=\frac{15}{28} \text{ and } \left(\frac{5}{7}\times\frac{3}{4}\right)=\frac{(5\times3)}{(7\times4)}=\frac{15}{28}$$

$$\therefore \left(\frac{3}{4}\times\frac{5}{7}\right)=\left(\frac{5}{7}\times\frac{3}{4}\right)$$

(ii) Let us consider the rational numbers $\frac{-2}{5}$ and $\frac{6}{7}$. Then,

$$\left(\frac{-2}{5}\times\frac{6}{7}\right)=\frac{(-2)\times6}{5\times7}=\frac{-12}{35} \text{ and } \left(\frac{6}{7}\times\frac{-2}{5}\right)=\frac{6\times(-2)}{7\times5}=\frac{-12}{35}$$

$$\therefore \left(\frac{-2}{5}\times\frac{6}{7}\right)=\left(\frac{6}{7}\times\frac{-2}{5}\right).$$

(iii) Let us consider the rational numbers $\frac{-2}{3}$ and $\frac{-5}{7}$. Then,

$$\left(\frac{-2}{3}\right)\times\left(\frac{-5}{7}\right)=\frac{(-2)\times(-5)}{3\times7}=\frac{10}{21} \text{ and } \left(\frac{-5}{7}\right)\times\left(\frac{-2}{3}\right)=\frac{(-5)\times(-2)}{7\times3}=\frac{10}{21}.$$

$$\therefore \left(\frac{-2}{3}\right)\times\left(\frac{-5}{7}\right)=\left(\frac{-5}{7}\right)\times\left(\frac{-2}{3}\right).$$

Property 3 (Associative Law): *While multiplying three or more rational numbers, they can be grouped in any order.*

Thus, for any rationals $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$, we have

$$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right).$$

EXAMPLE Consider the rationals $\frac{-5}{2}, \frac{-7}{4}$ and $\frac{1}{3}$. We have

$$\left(\frac{-5}{2}\times\frac{-7}{4}\right)\times\frac{1}{3}=\left\{\frac{(-5)\times(-7)}{2\times4}\times\frac{1}{3}\right\}=\left(\frac{35}{8}\times\frac{1}{3}\right)=\frac{(35\times1)}{(8\times3)}=\frac{35}{24}$$

$$\text{and } \frac{-5}{2}\times\left(\frac{-7}{4}\times\frac{1}{3}\right)=\frac{-5}{2}\times\frac{(-7)\times1}{4\times3}=\left(\frac{-5}{2}\times\frac{-7}{12}\right)=\frac{(-5)\times(-7)}{(2\times12)}=\frac{35}{24}.$$

$$\therefore \left(\frac{-5}{2}\times\frac{-7}{4}\right)\times\frac{1}{3}=\frac{-5}{2}\times\left(\frac{-7}{4}\times\frac{1}{3}\right).$$

Property 4 (Existence of Multiplicative Identity):

For any rational number $\frac{a}{b}$, *we have* $\left(\frac{a}{b}\times1\right)=\left(1\times\frac{a}{b}\right)=\frac{a}{b}$.

1 is called the ***multiplicative identity*** for rationals.

EXAMPLES (i) Consider the rational number $\frac{3}{4}$. Then, we have

$$\left(\frac{3}{4}\times1\right)=\left(\frac{3}{4}\times\frac{1}{1}\right)=\frac{(3\times1)}{(4\times1)}=\frac{3}{4} \text{ and } \left(1\times\frac{3}{4}\right)=\left(\frac{1}{1}\times\frac{3}{4}\right)=\frac{(1\times3)}{(1\times4)}=\frac{3}{4}.$$

$$\left(\frac{3}{4}\times 1\right)=\left(1\times\frac{3}{4}\right)=\frac{3}{4}$$

(ii) Consider the rational number $\frac{-9}{13}$. Then, we have

$$\left(\frac{-9}{13}\times 1\right)=\left(\frac{-9}{13}\times\frac{1}{1}\right)=\frac{(-9)\times 1}{13\times 1}=\frac{-9}{13} \text{ and } \left(1\times\frac{-9}{13}\right)=\left(\frac{1}{1}\times\frac{-9}{13}\right)=\frac{1\times(-9)}{1\times 13}=\frac{-9}{13}$$

$$\left(\frac{-9}{13}\times 1\right)=\left(1\times\frac{-9}{13}\right)=\frac{-9}{13}$$

Property 5 (Existence of Multiplicative Inverse): *Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.*

Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1.$

$\frac{b}{a}$ *is called the* ***reciprocal*** *of* $\frac{a}{b}$.

Clearly, zero has no reciprocal.
Reciprocal of 1 is 1 and the reciprocal of (−1) is (−1).

EXAMPLES (i) Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1.$

(ii) Reciprocal of $\frac{-8}{9}$ is $\frac{-9}{8}$, since $\left(\frac{-8}{9}\times\frac{-9}{8}\right)=\left(\frac{-9}{8}\times\frac{-8}{9}\right)=1.$

(iii) Reciprocal of −3 is $\frac{-1}{3}$, since

$$\left(-3\times\frac{-1}{3}\right)=\left(\frac{-3}{1}\times\frac{-1}{3}\right)=\frac{(-3)\times(-1)}{1\times 3}=\frac{3}{3}=1 \text{ and } \left(\frac{-1}{3}\times -3\right)=\left(\frac{-1}{3}\times\frac{-3}{1}\right)=\frac{(-1)\times(-3)}{3\times 1}=1.$$

REMARK We denote the reciprocal of $\frac{a}{b}$ by $\left(\frac{a}{b}\right)^{-1}$.

Clearly, $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$.

Property 6 (Distributive Law of Multiplication Over Addition): *For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have*

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right).$$

EXAMPLE Consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$. We have

$$\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\right)\times\left\{\frac{4+(-5)}{6}\right\}=\left(\frac{-3}{4}\right)\times\left(\frac{-1}{6}\right)=\frac{(-3)\times(-1)}{4\times 6}=\frac{3}{24}=\frac{1}{8}.$$

Again, $\left(\frac{-3}{4}\right)\times\frac{2}{3}=\frac{(-3)\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$ and $\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)=\frac{(-3)\times(-5)}{4\times 6}=\frac{15}{24}=\frac{5}{8}$.

$$\therefore \left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}=\left(\frac{-1}{2}+\frac{5}{8}\right)=\frac{(-4+5)}{8}=\frac{1}{8}.$$

Hence, $\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}$.

$\therefore \left(\frac{3}{4}\times 1\right)=\left(1\times\frac{3}{4}\right)=\frac{3}{4}.$

(ii) Consider the rational number $\frac{-9}{13}$. Then, we have

$$\left(\frac{-9}{13}\times 1\right)=\left(\frac{-9}{13}\times\frac{1}{1}\right)=\frac{(-9)\times 1}{13\times 1}=\frac{-9}{13} \text{ and } \left(1\times\frac{-9}{13}\right)=\left(\frac{1}{1}\times\frac{-9}{13}\right)=\frac{1\times(-9)}{1\times 13}=\frac{-9}{13}.$$

$$\therefore \left(\frac{-9}{13}\times 1\right)=\left(1\times\frac{-9}{13}\right)=\frac{-9}{13}.$$

Property 5 (Existence of Multiplicative Inverse): *Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.*

Thus, $\left(\frac{a}{b}\times\frac{b}{a}\right)=\left(\frac{b}{a}\times\frac{a}{b}\right)=1.$

$\frac{b}{a}$ *is called the* ***reciprocal*** *of* $\frac{a}{b}$.

Clearly, zero has no reciprocal.

Reciprocal of 1 is 1 and the reciprocal of (−1) is (−1).

EXAMPLES (i) Reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$, since $\left(\frac{5}{7}\times\frac{7}{5}\right)=\left(\frac{7}{5}\times\frac{5}{7}\right)=1.$

(ii) Reciprocal of $\frac{-8}{9}$ is $\frac{-9}{8}$, since $\left(\frac{-8}{9}\times\frac{-9}{8}\right)=\left(\frac{-9}{8}\times\frac{-8}{9}\right)=1.$

(iii) Reciprocal of −3 is $\frac{-1}{3}$, since

$$\left(-3\times\frac{-1}{3}\right)=\left(\frac{-3}{1}\times\frac{-1}{3}\right)=\frac{(-3)\times(-1)}{1\times 3}=\frac{3}{3}=1 \text{ and } \left(\frac{-1}{3}\times -3\right)=\left(\frac{-1}{3}\times\frac{-3}{1}\right)=\frac{(-1)\times(-3)}{3\times 1}=1.$$

REMARK We denote the reciprocal of $\frac{a}{b}$ by $\left(\frac{a}{b}\right)^{-1}$.

Clearly, $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}.$

Property 6 (Distributive Law of Multiplication Over Addition): *For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have*

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right).$$

EXAMPLE Consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$. We have

$$\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\right)\times\left\{\frac{4+(-5)}{6}\right\}=\left(\frac{-3}{4}\right)\times\left(\frac{-1}{6}\right)=\frac{(-3)\times(-1)}{4\times 6}=\frac{3}{24}=\frac{1}{8}.$$

Again, $\left(\frac{-3}{4}\right)\times\frac{2}{3}=\frac{(-3)\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$ and $\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)=\frac{(-3)\times(-5)}{4\times 6}=\frac{15}{24}=\frac{5}{8}.$

$$\therefore \left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}=\left(\frac{-1}{2}+\frac{5}{8}\right)=\frac{(-4+5)}{8}=\frac{1}{8}.$$

Hence, $\left(\frac{-3}{4}\right)\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left\{\left(\frac{-3}{4}\right)\times\frac{2}{3}\right\}+\left\{\left(\frac{-3}{4}\right)\times\left(\frac{-5}{6}\right)\right\}.$

Property 7 (Multiplicative Property of 0): *Every rational number multiplied with 0 gives 0.*

Thus, for any rational number $\frac{a}{b}$, we have: $\left(\frac{a}{b}\times 0\right)=\left(0\times\frac{a}{b}\right)=0.$

EXAMPLES (i) $\left(\frac{5}{18}\times 0\right)=\left(\frac{5}{18}\times\frac{0}{1}\right)=\frac{(5\times 0)}{(18\times 1)}=\frac{0}{18}=0$. Similarly, $\left(0\times\frac{5}{18}\right)=0.$

(ii) $\left(\frac{-12}{17}\times 0\right)=\left(\frac{-12}{17}\times\frac{0}{1}\right)=\frac{(-12)\times 0}{17\times 1}=\frac{0}{17}=0$. Similarly, $\left(0\times\frac{-12}{17}\right)=0.$

SOLVED EXAMPLES

EXAMPLE 1. *Find the reciprocal of each of the following:*

(i) 12 (ii) −8 (iii) $\frac{5}{16}$ (iv) $\frac{-14}{17}$

Solution (i) Reciprocal of 12 is $\frac{1}{12}$.

(ii) Reciprocal of −8 is $\frac{1}{-8}$, i.e., $\frac{-1}{8}$.

(iii) Reciprocal of $\frac{5}{16}$ is $\frac{16}{5}$.

(iv) Reciprocal of $\frac{-14}{17}$ is $\frac{17}{-14}$, i.e., $\frac{-17}{14}$.

EXAMPLE 2. *Verify that:*

(i) $\left(\frac{-3}{16}\times\frac{8}{15}\right)=\left(\frac{8}{15}\times\frac{-3}{16}\right)$ (ii) $\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$

(iii) $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$

Solution (i) LHS $=\left(\frac{-3}{16}\times\frac{8}{15}\right)=\frac{(-3)\times 8}{16\times 15}=\frac{-24}{240}=\frac{-1}{10}$.

RHS $=\left(\frac{8}{15}\times\frac{-3}{16}\right)=\frac{8\times(-3)}{15\times 16}=\frac{-24}{240}=\frac{-1}{10}$.

∴ LHS = RHS.

Hence, $\left(\frac{-3}{16}\times\frac{8}{15}\right)=\left(\frac{8}{15}\times\frac{-3}{16}\right)$.

(ii) LHS $=\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\frac{2}{3}\times\frac{6\times(-14)}{7\times 15}=\frac{2}{3}\times\frac{-84}{105}$

$=\frac{2}{3}\times\frac{-4}{5}=\frac{2\times(-4)}{3\times 5}=\frac{-8}{15}$.

RHS $=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}=\frac{(2\times 6)}{(3\times 7)}\times\frac{-14}{15}=\frac{12}{21}\times\frac{-14}{15}$

$=\frac{4}{7}\times\frac{-14}{15}=\frac{4\times(-14)}{(7\times 15)}=\frac{-56}{105}=\frac{-8}{15}$.

∴ LHS = RHS.

Hence, $\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$.

(iii) LHS $= \frac{5}{6} \times \left(\frac{-4}{5} + \frac{-7}{10}\right) = \frac{5}{6} \times \left\{\frac{(-8)+(-7)}{10}\right\} = \frac{5}{6} \times \frac{-15}{10}$

$= \frac{5}{6} \times \frac{-3}{2} = \frac{5 \times (-3)}{6 \times 2} = \frac{-15}{12} = \frac{-5}{4}$

RHS $= \left(\frac{5}{6} \times \frac{-4}{5}\right) + \left(\frac{5}{6} \times \frac{-7}{10}\right)$

$= \frac{5 \times (-4)}{6 \times 5} + \frac{5 \times (-7)}{6 \times 10} = \frac{-20}{30} + \frac{-35}{60} = \frac{-2}{3} + \frac{-7}{12}$

$= \frac{(-8)+(-7)}{12} = \frac{-15}{12} = \frac{-5}{4}$

$\therefore$ LHS = RHS.

Hence, $\frac{5}{6} \times \left(\frac{-4}{5} + \frac{-7}{10}\right) = \left(\frac{5}{6} \times \frac{-4}{5}\right) + \left(\frac{5}{6} \times \frac{-7}{10}\right)$.

EXERCISE 1D

1. Find each of the following products:

(i) $\frac{3}{5} \times \frac{-7}{8}$ (ii) $\frac{-9}{2} \times \frac{5}{4}$ (iii) $\frac{-6}{11} \times \frac{-5}{3}$

(iv) $\frac{-2}{3} \times \frac{6}{7}$ (v) $\frac{-12}{5} \times \frac{10}{-3}$ (vi) $\frac{25}{-9} \times \frac{3}{-10}$

(vii) $\frac{5}{-18} \times \frac{-9}{20}$ (viii) $\frac{-13}{15} \times \frac{-25}{26}$ (ix) $\frac{16}{-21} \times \frac{14}{5}$

(x) $\frac{-7}{6} \times 24$ (xi) $\frac{7}{24} \times (-48)$ (xii) $\frac{-13}{5} \times (-10)$

2. Verify each of the following:

(i) $\frac{3}{7} \times \frac{-5}{9} = \frac{-5}{9} \times \frac{3}{7}$ (ii) $\frac{-8}{7} \times \frac{13}{9} = \frac{13}{9} \times \frac{-8}{7}$

(iii) $\frac{-12}{5} \times \frac{7}{-36} = \frac{7}{-36} \times \frac{-12}{5}$ (iv) $-8 \times \frac{-13}{12} = \frac{-13}{12} \times (-8)$

3. Verify each of the following:

(i) $\left(\frac{5}{7} \times \frac{12}{13}\right) \times \frac{7}{18} = \frac{5}{7} \times \left(\frac{12}{13} \times \frac{7}{18}\right)$ (ii) $\frac{-13}{24} \times \left(\frac{-12}{5} \times \frac{35}{36}\right) = \left(\frac{-13}{24} \times \frac{-12}{5}\right) \times \frac{35}{36}$

(iii) $\left(\frac{-9}{5} \times \frac{-10}{3}\right) \times \frac{21}{-4} = \frac{-9}{5} \times \left(\frac{-10}{3} \times \frac{21}{-4}\right)$

4. Fill in the blanks:

(i) $\frac{-23}{17} \times \frac{18}{35} = \frac{18}{35} \times$ (......) (ii) $-38 \times \frac{-7}{19} = \frac{-7}{19} \times$ (......)

(iii) $\left(\frac{15}{7} \times \frac{-21}{10}\right) \times \frac{-5}{6} =$ (......) $\times \left(\frac{-21}{10} \times \frac{-5}{6}\right)$ (iv) $\frac{-12}{5} \times \left(\frac{4}{15} \times \frac{25}{-16}\right) = \left(\frac{-12}{5} \times \frac{4}{15}\right) \times$ (......)

5. Find the multiplicative inverse (i.e., reciprocal) of:

(i) $\frac{13}{25}$ (ii) $\frac{-17}{12}$ (iii) $\frac{-7}{24}$ (iv) 18 (v) −16

(vi) $\frac{-3}{-5}$ (vii) −1 (viii) $\frac{0}{2}$ (ix) $\frac{2}{-5}$ (x) $\frac{-1}{8}$

(iii) LHS $= \frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right) = \frac{5}{6}\times\left[\frac{(-8)+(-7)}{10}\right] = \frac{5}{6}\times\frac{-15}{10}$

$= \frac{5}{6}\times\frac{-3}{2} = \frac{5\times(-3)}{6\times 2} = \frac{-15}{12} = \frac{-5}{4}$

RHS $= \left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$

$= \frac{5\times(-4)}{6\times 5}+\frac{5\times(-7)}{6\times 10} = \frac{-20}{30}+\frac{-35}{60} = \frac{-2}{3}+\frac{-7}{12}$

$= \frac{(-8)+(-7)}{12} = \frac{-15}{12} = \frac{-5}{4}.$

$\therefore$ LHS = RHS.

Hence, $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right) = \left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right).$

EXERCISE 1D

1. Find each of the following products:

(i) $\frac{3}{5}\times\frac{-7}{8}$ (ii) $\frac{-9}{2}\times\frac{5}{4}$ (iii) $\frac{-6}{11}\times\frac{-5}{3}$

(iv) $\frac{-2}{3}\times\frac{6}{7}$ (v) $\frac{-12}{5}\times\frac{10}{-3}$ (vi) $\frac{25}{-9}\times\frac{3}{-10}$

(vii) $\frac{5}{-18}\times\frac{-9}{20}$ (viii) $\frac{-13}{15}\times\frac{-25}{26}$ (ix) $\frac{16}{-21}\times\frac{14}{5}$

(x) $\frac{-7}{6}\times 24$ (xi) $\frac{7}{24}\times(-48)$ (xii) $\frac{-13}{5}\times(-10)$

2. Verify each of the following:

(i) $\frac{3}{7}\times\frac{-5}{9}=\frac{-5}{9}\times\frac{3}{7}$ (ii) $\frac{-8}{7}\times\frac{13}{9}=\frac{13}{9}\times\frac{-8}{7}$

(iii) $\frac{-12}{5}\times\frac{7}{-36}=\frac{7}{-36}\times\frac{-12}{5}$ (iv) $-8\times\frac{-13}{12}=\frac{-13}{12}\times(-8)$

3. Verify each of the following:

(i) $\left(\frac{5}{7}\times\frac{12}{13}\right)\times\frac{7}{18}=\frac{5}{7}\times\left(\frac{12}{13}\times\frac{7}{18}\right)$ (ii) $\frac{-13}{24}\times\left(\frac{-12}{5}\times\frac{35}{36}\right)=\left(\frac{-13}{24}\times\frac{-12}{5}\right)\times\frac{35}{36}$

(iii) $\left(\frac{-9}{5}\times\frac{-10}{3}\right)\times\frac{21}{-4}=\frac{-9}{5}\times\left(\frac{-10}{3}\times\frac{21}{-4}\right)$

4. Fill in the blanks:

(i) $\frac{-23}{17}\times\frac{18}{35}=\frac{18}{35}\times(\ldots\ldots)$ (ii) $-38\times\frac{-7}{19}=\frac{-7}{19}\times(\ldots\ldots)$

(iii) $\left(\frac{15}{7}\times\frac{-21}{10}\right)\times\frac{-5}{6}=(\ldots\ldots)\times\left(\frac{-21}{10}\times\frac{-5}{6}\right)$ (iv) $\frac{-12}{5}\times\left(\frac{4}{15}\times\frac{25}{-16}\right)=\left(\frac{-12}{5}\times\frac{4}{15}\right)\times(\ldots\ldots)$

5. Find the multiplicative inverse (i.e., reciprocal) of:

(i) $\frac{13}{25}$ (ii) $\frac{-17}{12}$ (iii) $\frac{-7}{24}$ (iv) 18 (v) −16

(vi) $\frac{-3}{-5}$ (vii) −1 (viii) $\frac{0}{2}$ (ix) $\frac{2}{-5}$ (x) $\frac{-1}{8}$

6. Find the value of:

(i) $\left(\frac{5}{8}\right)^{-1}$ (ii) $\left(\frac{-4}{9}\right)^{-1}$ (iii) $(-7)^{-1}$ (iv) $\left(\frac{1}{-3}\right)^{-1}$

7. Verify the following:

(i) $\frac{3}{7}\times\left(\frac{5}{6}+\frac{12}{13}\right)=\left(\frac{3}{7}\times\frac{5}{6}\right)+\left(\frac{3}{7}\times\frac{12}{13}\right)$

(ii) $\frac{-15}{4}\times\left(\frac{3}{7}+\frac{-12}{5}\right)=\left(\frac{-15}{4}\times\frac{3}{7}\right)+\left(\frac{-15}{4}\times\frac{-12}{5}\right)$

(iii) $\left(\frac{-8}{3}+\frac{-13}{12}\right)\times\frac{5}{6}=\left(\frac{-8}{3}\times\frac{5}{6}\right)+\left(\frac{-13}{12}\times\frac{5}{6}\right)$

(iv) $\frac{-16}{7}\times\left(\frac{-8}{9}+\frac{-7}{6}\right)=\left(\frac{-16}{7}\times\frac{-8}{9}\right)+\left(\frac{-16}{7}\times\frac{-7}{6}\right)$

8. Name the property of multiplication illustrated by each of the following statements:

(i) $\frac{-15}{8}\times\frac{-12}{7}=\frac{-12}{7}\times\frac{-15}{8}$

(ii) $\left(\frac{-2}{3}\times\frac{7}{9}\right)\times\frac{-9}{5}=\frac{-2}{3}\times\left(\frac{7}{9}\times\frac{-9}{5}\right)$

(iii) $\frac{-3}{4}\times\left(\frac{-5}{6}+\frac{7}{8}\right)=\left(\frac{-3}{4}\times\frac{-5}{6}\right)+\left(\frac{-3}{4}\times\frac{7}{8}\right)$

(iv) $\frac{-16}{9}\times 1=1\times\frac{-16}{9}=\frac{-16}{9}$

(v) $\frac{-11}{15}\times\frac{15}{-11}=\frac{15}{-11}\times\frac{-11}{15}=1$

(vi) $\frac{-7}{5}\times 0=0$

9. Fill in the blanks:

(i) The product of a rational number and its reciprocal is ....
(ii) Zero has ...... reciprocal.
(iii) The numbers ...... and ...... are their own reciprocals.
(iv) Zero is ...... the reciprocal of any number.
(v) The reciprocal of a, where $a \neq 0$, is ....
(vi) The reciprocal of $\frac{1}{a}$, where $a \neq 0$, is ....
(vii) The reciprocal of a positive rational number is ....
(viii) The reciprocal of a negative rational number is ....

DIVISION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, we define, $\left(\frac{a}{b}\div\frac{c}{d}\right)=\left(\frac{a}{b}\times\frac{d}{c}\right)$.

When $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is called the ***dividend;*** $\frac{c}{d}$ is called the ***divisor*** and the result is known as ***quotient.***

SOLVED EXAMPLES

EXAMPLE 1. *Divide:*

(i) $\frac{9}{16}$ *by* $\frac{5}{8}$ (ii) $\frac{-6}{25}$ by $\frac{3}{5}$ (iii) $\frac{11}{24}$ *by* $\frac{-5}{8}$ (iv) $\frac{-9}{40}$ *by* $\frac{-3}{8}$

Solution We have:

(i) $\frac{9}{16} \div \frac{5}{8} = \frac{9}{16} \times \frac{8}{5} = \frac{9 \times 8}{16 \times 5} = \frac{72}{80} = \frac{9}{10}$.

(ii) $\frac{-6}{25} \div \frac{3}{5} = \frac{-6}{25} \times \frac{5}{3} = \frac{(-6) \times 5}{25 \times 3} = \frac{-30}{75} = \frac{-2}{5}$.

(iii) $\frac{11}{24} \div \frac{-5}{8} = \frac{11}{24} \times \frac{8}{-5} = \frac{11 \times 8}{24 \times (-5)} = \frac{88}{-120} = \frac{-11}{15}$.

(iv) $\frac{-9}{40} \div \frac{-3}{8} = \frac{-9}{40} \times \frac{8}{-3} = \frac{(-9) \times 8}{40 \times (-3)} = \frac{-72}{-120} = \frac{3}{5}$.

EXAMPLE 2. *The product of two numbers is* $\frac{-28}{27}$. *If one of the numbers is* $\frac{-4}{9}$, *find the other.*

Solution Let the other number be x. Then,

$$x \times \frac{-4}{9} = \frac{-28}{27}$$

$$\Rightarrow x = \frac{-28}{27} \div \frac{-4}{9} = \frac{-28}{27} \times \frac{9}{-4} = \frac{(-28) \times 9}{27 \times (-4)} = \frac{-(28 \times 9)}{-(27 \times 4)}$$

$$\Rightarrow x = \frac{\cancel{28}^{7} \times \cancel{9}^{1}}{\cancel{27}_{3} \times \cancel{4}_{1}} = \frac{7}{3}.$$

Hence, the other number is $\frac{7}{3}$.

EXAMPLE 3. *Fill in the blanks:* $\frac{27}{16} \div (\ldots\ldots) = \frac{-15}{8}$.

Solution Let $\frac{27}{16} \div \left(\frac{a}{b}\right) = \frac{-15}{8}$. Then,

$$\frac{27}{16} \times \frac{b}{a} = \frac{-15}{8} \Rightarrow \frac{b}{a} = \frac{-15}{8} \times \frac{16}{27} = \frac{-10}{9}$$

$$\Rightarrow \frac{a}{b} = \frac{9}{-10} = \frac{-9}{10}.$$

Hence, the missing number is $\frac{-9}{10}$.

PROPERTIES OF DIVISION

Property 1 (Closure Property): *If* $\frac{a}{b}$ *and* $\frac{c}{d}$ *are any two rational numbers such that* $\frac{c}{d} \neq 0$ *then* $\left(\frac{a}{b} \div \frac{c}{d}\right)$ *is also a rational number.*

Property 2 (Property of 1): *For every rational number* $\frac{a}{b}$, *we have:*

$$\left(\frac{a}{b} \div 1\right) = \frac{a}{b}.$$

Property 3: *For every nonzero rational number* $\frac{a}{b}$, *we have:*

$$\left(\frac{a}{b} \div \frac{a}{b}\right) = 1.$$

EXERCISE 1E

1. Simplify:

   (i) $\frac{4}{9} \div \frac{-5}{12}$ (ii) $-8 \div \frac{-7}{16}$ (iii) $\frac{-12}{7} \div (-18)$

   (iv) $\frac{-1}{10} \div \frac{-8}{5}$ (v) $\frac{-16}{35} \div \frac{-15}{14}$ (vi) $\frac{-65}{14} \div \frac{13}{7}$

2. Verify whether the given statement is true or false:

   (i) $\frac{13}{5} \div \frac{26}{10} = \frac{26}{10} \div \frac{13}{5}$ (ii) $-9 \div \frac{3}{4} = \frac{3}{4} \div (-9)$

   (iii) $\frac{-8}{9} \div \frac{-4}{3} = \frac{-4}{3} \div \frac{-8}{9}$ (iv) $\frac{-7}{24} \div \frac{3}{-16} = \frac{3}{-16} \div \frac{-7}{24}$

3. Verify whether the given statement is true or false:

   (i) $\left(\frac{5}{9} \div \frac{1}{3}\right) \div \frac{5}{2} = \frac{5}{9} \div \left(\frac{1}{3} \div \frac{5}{2}\right)$ (ii) $\left\{(-16) \div \frac{6}{5}\right\} \div \frac{-9}{10} = (-16) \div \left\{\frac{6}{5} \div \frac{-9}{10}\right\}$

   (iii) $\left(\frac{-3}{5} \div \frac{-12}{35}\right) \div \frac{1}{14} = \frac{-3}{5} \div \left(\frac{-12}{35} \div \frac{1}{14}\right)$

4. The product of two rational numbers is -9. If one of the numbers is -12, find the other.

5. The product of two rational numbers is $\frac{-16}{9}$. If one of the numbers is $\frac{-4}{3}$, find the other.

6. By what rational number should we multiply $\frac{-15}{56}$ to get $\frac{-5}{7}$?

7. By what rational number should $\frac{-8}{39}$ be multiplied to obtain $\frac{1}{26}$?

8. By what number should $\frac{-33}{8}$ be divided to get $\frac{-11}{2}$?

9. Divide the sum of $\frac{13}{5}$ and $\frac{-12}{7}$ by the product of $\frac{-31}{7}$ and $\frac{1}{-2}$.

10. Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.

11. Fill in the blanks:

    (i) $\frac{9}{8} \div (\ldots\ldots) = \frac{-3}{2}$ (ii) $(\ldots\ldots) \div \left(\frac{-7}{5}\right) = \frac{10}{19}$

    (iii) $(\ldots\ldots) \div (-3) = \frac{-4}{15}$ (iv) $(-12) \div (\ldots\ldots) = \frac{-6}{5}$

12. (i) Are rational numbers always closed under division?

    (ii) Are rational numbers always commutative under division?

    (iii) Are rational numbers always associative under division?

    (iv) Can we divide 1 by 0?

An Important Result

If x and y be two rational numbers such that $x < y$ then $\frac{1}{2}(x + y)$ is a rational number between x and y.

EXAMPLE 1. *Find a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.*

Solution Required number $= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right)$

$$= \frac{1}{2}\left(\frac{2+3}{6}\right) = \left(\frac{1}{2} \times \frac{5}{6}\right) = \frac{5}{12}$$

Hence, $\frac{5}{12}$ is a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

EXAMPLE 2. *Find three rational numbers lying between 3 and 4.*

Solution A rational number between 3 and 4 is $\frac{1}{2}(3 + 4) = \frac{7}{2}$

Then, $3 < \frac{7}{2} < 4$.

A rational number between 3 and $\frac{7}{2} = \frac{1}{2}\left(3 + \frac{7}{2}\right) = \frac{1}{2}\left(\frac{3}{1} + \frac{7}{2}\right)$

$$= \frac{1}{2}\left(\frac{6+7}{2}\right) = \left(\frac{1}{2} \times \frac{13}{2}\right) = \frac{13}{4}.$$

A rational number between $\frac{7}{2}$ and $4 = \frac{1}{2}\left(\frac{7}{2} + 4\right) = \frac{1}{2}\left(\frac{7}{2} + \frac{4}{1}\right)$

$$= \frac{1}{2}\left(\frac{7+8}{2}\right) = \left(\frac{1}{2} \times \frac{15}{2}\right) = \frac{15}{4}.$$

$\therefore \quad 3 < \frac{13}{4} < \frac{7}{2} < \frac{15}{4} < 4.$

Hence, the required numbers are $\frac{13}{4}$, $\frac{7}{2}$ and $\frac{15}{4}$.

ALTERNATIVE METHOD OF FINDING LARGE NUMBER OF RATIONAL NUMBERS BETWEEN TWO GIVEN RATIONAL NUMBERS

EXAMPLE 3. *Find 20 rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.*

Solution LCM of 6 and 8 is 24.

Now, $\frac{-5}{6} = \frac{-5 \times 4}{6 \times 4} = \frac{-20}{24}$ and $\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$.

Rational numbers lying between $\frac{-5}{6}$ and $\frac{5}{8}$ are

$$\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \frac{-16}{24}, \ldots, \frac{-1}{24}, \frac{0}{24}, \frac{1}{24}, \frac{2}{24}, \frac{3}{24}, \ldots, \frac{14}{24}.$$

Out of these 20 may be taken.

EXAMPLE 4. *Find 15 rational numbers between −2 and 0.*

Solution We may write, $-2 = \frac{-20}{10}$ and $0 = \frac{0}{10}$.

Rational numbers lying between −2 and 0 are

$\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \frac{-14}{10}, \frac{-13}{10}, \frac{-12}{10}, \frac{-11}{10}, -1, \frac{-9}{10},$

$\frac{-8}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}, \frac{-4}{10}, \frac{-3}{10}, \frac{-2}{10}, \frac{-1}{10}.$

Out of these 15 may be taken.

EXAMPLE 5. *Write 9 rational numbers between 1 and 2.*

Solution We may write $1 = \frac{10}{10}$ and $2 = \frac{20}{10}$

$\therefore$ rational numbers between 1 and 2 are

$\frac{11}{10}, \frac{12}{10}, \frac{13}{10}, \frac{14}{10}, \frac{15}{10}, \frac{16}{10}, \frac{17}{10}, \frac{18}{10}, \frac{19}{10}.$

REMARK *Suppose we have to write 99 rational numbers between 1 and 2.*

Then, we may write, $1 = \frac{100}{100}$ and $2 = \frac{200}{100}$.

$\therefore$ rational numbers between 1 and 2 are

$\frac{101}{100}, \frac{102}{100}, \frac{103}{100}, \ldots, \frac{198}{100}, \frac{199}{100}.$

EXERCISE 1F

1. Find a rational number between $\frac{1}{4}$ and $\frac{1}{3}$.
2. Find a rational number between 2 and 3.
3. Find a rational number between $\frac{-1}{3}$ and $\frac{1}{2}$.
4. Find two rational numbers between –3 and –2.
5. Find three rational numbers between 4 and 5.
6. Find three rational numbers between $\frac{2}{3}$ and $\frac{3}{4}$.
7. Find 10 rational numbers between $\frac{-3}{4}$ and $\frac{5}{6}$.
8. Find 12 rational numbers between –1 and 2.

WORD PROBLEMS

EXERCISE 1G

1. From a rope 11 m long, two pieces of lengths $2\frac{3}{5}$ m and $3\frac{3}{10}$ m are cut off. What is the length of the remaining rope?
2. A drum full of rice weighs $40\frac{1}{6}$ kg. If the empty drum weighs $13\frac{3}{4}$ kg, find the weight of rice in the drum.
3. A basket contains three types of fruits weighing $19\frac{1}{3}$ kg in all. If $8\frac{1}{9}$ kg of these be apples, $3\frac{1}{6}$ kg be oranges and the rest pears, what is the weight of the pears in the basket?

4. On one day a rickshaw puller earned ₹ 160. Out of his earnings he spent ₹ $26\frac{3}{5}$ on tea and snacks, ₹ $50\frac{1}{2}$ on food and ₹ $16\frac{2}{5}$ on repairs of the rickshaw. How much did he save on that day?

5. Find the cost of $3\frac{2}{5}$ metres of cloth at ₹ $63\frac{3}{4}$ per metre.

6. A car is moving at an average speed of $60\frac{2}{5}$ km/hr. How much distance will it cover in $6\frac{1}{4}$ hours?

7. Find the area of a rectangular park which is $36\frac{3}{5}$ m long and $16\frac{2}{3}$ m broad.

8. Find the area of a square plot of land whose each side measures $8\frac{1}{2}$ metres.

9. One litre of petrol costs ₹ $63\frac{3}{4}$. What is the cost of 34 litres of petrol?

10. An aeroplane covers 1020 km in an hour. How much distance will it cover in $4\frac{1}{6}$ hours?

11. The cost of $3\frac{1}{2}$ metres of cloth is ₹ $166\frac{1}{4}$. What is the cost of one metre of cloth?

12. A cord of length $71\frac{1}{2}$ m has been cut into 26 pieces of equal length. What is the length of each piece?

13. The area of a room is $65\frac{1}{4}$ m^2. If its breadth is $5\frac{7}{16}$ metres, what is its length?

14. The product of two fractions is $9\frac{3}{5}$. If one of the fractions is $9\frac{3}{7}$, find the other.

15. In a school, $\frac{5}{8}$ of the students are boys. If there are 240 girls, find the number of boys in the school.

16. After reading $\frac{7}{9}$ of a book, 40 pages are left. How many pages are there in the book?

17. Rita had ₹ 300. She spent $\frac{1}{3}$ of her money on notebooks and $\frac{1}{4}$ of the remainder on stationery items. How much money is left with her?

18. Amit earns ₹ 32000 per month. He spends $\frac{1}{4}$ of his income on food; $\frac{3}{10}$ of the remainder on house rent and $\frac{5}{21}$ of the remainder on the education of children. How much money is still left with him?

19. If $\frac{3}{5}$ of a number exceeds its $\frac{2}{7}$ by 44, find the number.

20. At a cricket test match $\frac{2}{7}$ of the spectators were in a covered place while 15000 were in open. Find the total number of spectators.

EXERCISE 1H

OBJECTIVE QUESTIONS

Tick (✓) the correct answer in each of the following:

1. $\left(\frac{-5}{16}+\frac{7}{12}\right)=?$

(a) $-\frac{7}{48}$ (b) $\frac{1}{24}$ (c) $\frac{13}{48}$ (d) $\frac{1}{3}$